Use of Pipe Flow Correlations to Predict Turbulent Skin Friction for Drag-Reducing Fluids

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Experimental data are used to determine the effective limits of the Meyer correlation for interpreting turbulent pipe flow experiments with drag-reducing additives. Definition of these limits is necessary in order to design pipe flow experiments suitable for measuring the "universal" drag-reducing properties of these fluids. Data from various sources for 10 different pipe sizes are used in the study. The results show that the correlation is suitable for generalization if the ratio of the velocity profile shift function ΔB to the dimensionless pipe radius η_R is greater than 0.033. This specifies a minimum pipe size to be used for a given additive solution to obtain results which can be generalized. This minimum pipe diameter effect, first suggested by Granville, can be used to explain anomalous drag reduction measurements in small pipes and thin boundary layers. The results of this study also suggest that degradation of the additive solution occurs in single-pass pipe flow systems at a given friction velocity for a particular solution, independent of pipe length. In addition, degradation occurs in poly(ethylene oxide) solutions at about the same value of friction velocity as in acrylamide copolymers. The basic correlation is suitable for the solutions studied up to a friction velocity of about 1 fps, when degradation occurs.

Introduction

THERE has been considerable interest recently in predicting effects of drag-reducing polymer additives on turbulent skin friction of bodies moving through fluids. However, very little is known experimentally, particularly for high Reynolds number flows. Indeed, most information on the effects of additives on various aspects of turbulent skin friction has been obtained in pipe flow. It is an economic advantage and a scientific convenience to use data from pipe flows to predict the flow in external boundary layers. Analytical extension from pipe flow to boundary-layer flow is relatively straightforward for Newtonian fluids, but is apparently not for very effective drag-reducing fluids. So many exceptions have been found to the original correlations for moderately effective additives that some investigators have suggested that the more effective additives behave differently. The results of this investigation show that they do not.

The purpose of this study is to determine the effective limits of the Meyer correlation for interpreting pipe flow experiments. Consistent with that purpose is the view that the only key to successful correlation of skin-friction data is the behavior of the velocity profile in the presence of various additives.

Meyer[1] used velocity profile data to correlate pipe flow skin-friction measurements with flow parameters, because a semilogarithmic plot of the velocity profile data for drag-reducing fluids showed a parallel shift compared to a similar plot for profiles taken in water. The shift occurred only above a critical shear stress, or friction velocity, and was a function only of friction velocity, independent of pipe diameter. Thus, the skin-friction relation could be developed from the velocity profile evaluated at the centerline, as for Newtonian fluids, carrying through the profile shift ΔB. ΔB was defined by Meyer in terms of a semilogarithmic function involving two empirical fluid properties, α and u_{*cr}, which could be determined directly from pressure drop-flow rate measurements without velocity profile measurements. The only condition for the use of the correlation is the simple shift of the velocity profile. The utility of the correlation is that it is based on a universal velocity profile, as demonstrated for Newtonian fluids, and may be used to develop correlations for other turbulent shear flows, such as external boundary layers.

This correlation was used by Granville[2] to derive an expression for predicting skin friction on flat plates in drag-reducing fluids. It was later used by White[3] and Wells[4] in similar flat plate analyses. Thus, the prediction of skin friction on flat plates is based on the determination of α and u_{*cr} from simple pipe flow experiments.

An examination of difficulties in interpreting the correlation in terms of the pipe flow data of several investigators is the substance of this paper. These are divided into two categories: 1) the use of pipe sizes that are too small to permit full development (in the transverse direction, not in the streamwise direction) of the semilogarithmic velocity profile, thus making it inappropriate to interpret friction factor Reynolds number data in terms of a simple shift of the velocity profile; and 2) the deviation of friction factor data from the correlation at high shear velocities, apparently due to degradation of the polymer molecules.

The "minimum pipe diameter" effect was first discussed by Granville. It is a limitation inherent in turbulent flow of Newtonian fluids in small pipes at low Reynolds numbers, but has a more important effect in polymer solutions where the sublayer and transitional layers are increased in thickness. Virk[7] proposed a universal maximum friction reduction limit for relatively small pipes and relatively large additive concentrations. Friction reduction beyond this limit was not seen in his data. Although this proposed lower limit may exist, nongeneral data may be generated in small pipes before it is reached, e.g., data for 50 ppm AP-30 given in Ref. 8. Therefore, in this study in which the limits on the Meyer correlation are desired, the data will be examined to determine the conditions that prevent generalizable data to be obtained in small pipes.

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Table 1 A summary of α and u_{*cr} values determined from data in large pipes, and degradation limits

Fluid	α	u_{*cr}, fps	Degradation limit, u_*, fps	Ref.
P-295, 10 ppm	4.9	0.18	>2.4	6
P-295, 18 ppm	12.8	0.115	1.0	6
P-295, 31 ppm	16.8	0.12	1.0	6, 9
P-295, 56 ppm[a]	23.6	0.086	1.1	6
P-295, 100 ppm	23.8	0.057	>1.5	6, 9
AP-30, 50 ppm	18.5	0.080	0.8	8
AP-30, 100 ppm	22.3	0.052	0.7	8
WSR-301, 5 ppm	21.2	0.098	>1.0	7
WSR-301, 20 ppm	21.8	0.087	>1.0	7
WSR-301, 100 ppm[a]	36.0	0.057	>0.8	7

[a] Correlation determined from data for one pipe only.

It is shown by Fabula[5] and Wells[4] that the minimum shear layer thickness is not significant on large, fast vehicles, since an appreciable part of the boundary layer on that type of body is thick enough to be beyond the limit. The problem lies primarily in the interpretation of pipe flow data that have been taken in pipes having diameters too small to fulfill the condition of the Meyer correlation, with the practical effect that experiments in small pipes yield friction reduction less than actually attainable in shear layers of greater thickness.

The second difficulty in the interpretation of pipe flow data, i.e., degradation of the polymer solution, has been identified by a number of investigators, but has not been specifically investigated. The causes of degradation are not known, although there is evidence that high shear stresses are at least partly responsible. Therefore, as Fabula[5] points out, pipe flow data must be taken at sufficiently high shear velocities to duplicate that of the flow to be predicted. The results of this study show that the correlation of Meyer can be used accurately over an appreciable ring of shear velocities of interest. The results also suggest that degradation is not a function of dwell time in the turbulent shear flow, but this point must be investigated further.

Description of Experiments

Since the purpose of this study is to determine the effective limits of the basic correlation, it was necessary to find experimental data for a given fluid that provided: 1) valid measurements of the drag-reducing properties and 2) cases for which the aforementioned limits were exceeded. This requires data for more than one pipe size over a significant range of friction velocities. Three such sets of data were found. First, experiments with solutions of Polyhall P-295, a copolymer of acrylamide and acrylic acid (a product of the Stein Hall Company) have been performed in this laboratory recently.[6] Second, the tests by Virk[7] with Polyox WSR-301, poly(ethylene oxide) (a product of Union Carbide Company), were satisfactory, although Virk's larger pipe size was used in a recirculating system and subject to continuous degradation due to pumping (all other data used in this study were obtained in single-pass, or blow-down, systems). Third, recent data of Whitsitt et al.[8] for Separan AP-30 (a copolymer product of the Dow Chemical Company) were used. The details of the facilities and the experimental procedures are given in Refs. 7 and 8 for the Polyox and AP-30 solutions, respectively, and in Refs. 6 and 9 for the Polyhall solutions. Some data in all three papers were not used in this study because either there were not sufficient data in a relatively large pipe to permit α and u_{*cr} to be accurately determined, or the data for all pipe sizes correlated, indicating that the minimum pipe diameter was not exceeded. Data for Polyhall solutions for which the latter was true are summarized in Table 1.

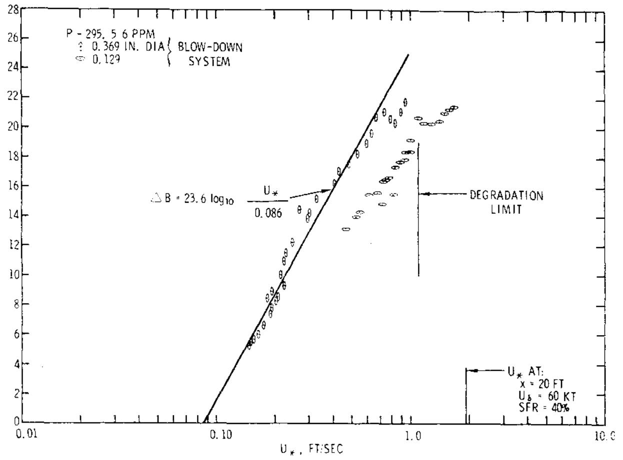

Fig. 1 Pipe flow friction data for a 56 ppm P-295 solution in two pipe sizes.

Analysis of Data

The form of the shift of the semilogarithmic velocity profile due to drag-reducing additives given by Meyer[1] is

$$\Delta B = \alpha \log_{10}(u_*/u_{*cr}) \qquad (u_* \geq u_{*cr}) \tag{1}$$

where ΔB is the parallel shift in the dimensionless velocity u/u_*, u is the local velocity, u_* is the friction velocity $(\tau_0/\rho)^{1/2}$, τ_0 is shear stress at the wall, ρ is fluid density, α is a constant of proportionality, a property of the fluid (a given concentration of a given additive), and u_{*cr} is the critical friction velocity for the onset of friction reduction. Thus the velocity profile could be written

$$\frac{u}{u_*} = 5.77 \log_{10} \frac{yu_*}{\nu_w} + 5.5 + \alpha \log_{10} \frac{u_*}{u_{*cr}} \quad (u_* \geq u_{*cr}) \tag{2}$$

where y is distance from the wall and ν_w is kinematic viscosity at the wall. Equation (2) was evaluated at the pipe centerline, the relation between the average velocity and the centerline velocity for Newtonian fluids was inserted, and the constants were changed slightly to reduce to the relation for Newtonian fluids, which gives

$$\frac{1}{(f)^{1/2}} = 4 \log_{10} Re(f)^{1/2} - 0.394 + \frac{\Delta B}{(2)^{1/2}} \quad (u_* \geq u_{*cr}) \tag{3}$$

where $f = 2u_*^2/\bar{u}^2$, $Re = \bar{u}D/\nu_w$, $\bar{u}$ is bulk average velocity in the pipe, and D is pipe diameter. Thus Eq. (3) can be rewritten as

$$\Delta B = (2)^{1/2}[1/(f)^{1/2} - 4 \log_{10} Re(f)^{1/2} + 0.394] \quad (u_* \geq u_{*cr}) \tag{4}$$

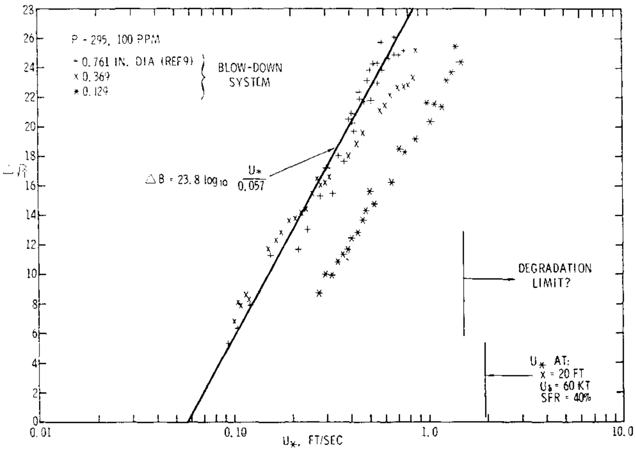

Fig. 2 Pipe flow friction data for a 100 ppm P-295 solution in three pipe sizes.

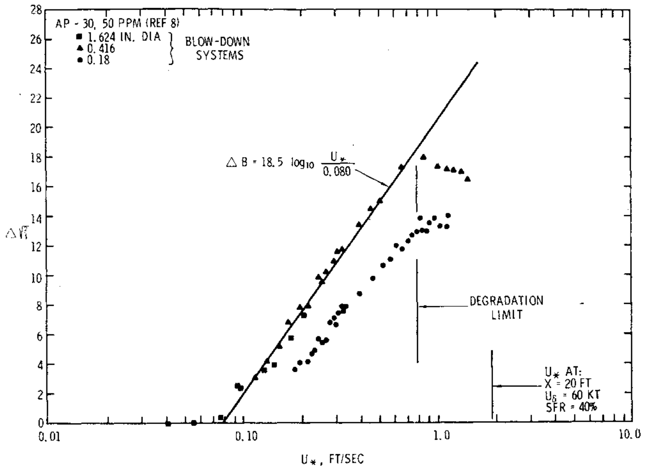

Fig. 3 Pipe flow friction data for a 50-ppm AP-30 solution in three pipe sizes.

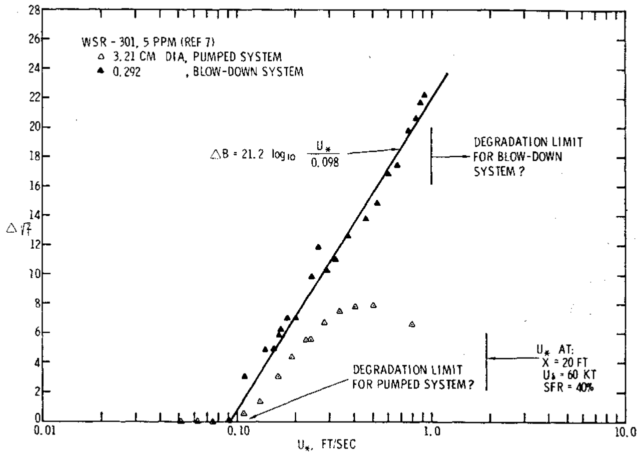

Fig. 5 Pipe flow friction data for a 5 ppm WSR-301 solution in two pipe sizes.

and used to determine α and u_{*cr} from friction factor Reynolds number data for pipe flow, if the velocity profile is of the form of Eq. (2). If the velocity profile is not of that form, as seen in several investigations, e.g., Refs. 10 and 11, then Eq. (4) has no basis. But since velocity profile data are not available to evaluate Eq. (2), it is useful to examine friction factor data in terms of the form of Eq. (3), which resulted from Eq. (2). Therefore, for purposes of this study a general function is defined to give the shift in $(2/f)^{1/2}$, at a given value of $Re(f)^{1/2}$, from the corresponding Newtonian value

$$\Delta(2/f)^{1/2} \equiv (2/f)^{1/2} - (2/f_N)^{1/2} \qquad (5)$$

where f is the measured friction factor for drag-reducing fluids and f_N is the friction factor for Newtonian fluids, which is given by

$$1/(f_N)^{1/2} = 4 \log_{10} Re(f_N)^{1/2} - 0.394 \qquad (6)$$

Equation (5) is written to make a distinction in nomenclature between ΔB and $\Delta(2/f)^{1/2}$. The two Δ functions will be identical when the velocity profile is of the form of Eq. (2) [i.e., ΔB will represent the parallel shift in the profile, and will be equal to $\Delta(2/f)^{1/2}$], but when Eq. (2) does not apply, as in the case of pipe diameters which are too small, the shift in friction factor is not a simple shift in the velocity profile and the term ΔB is not appropriate. The question to which the data will be put is whether the $\Delta(2/f)^{1/2}$ function is a function only of u_*, as is the ΔB function.

The appropriate data from Refs. 6–9 have been replotted as $\Delta(2/f)^{1/2}$ vs u_* in Figs. 1–7, using Eq. (5). The data for the largest pipe size for each fluid have been fitted to Eq. (1) from which α and u_{*cr} have been determined.

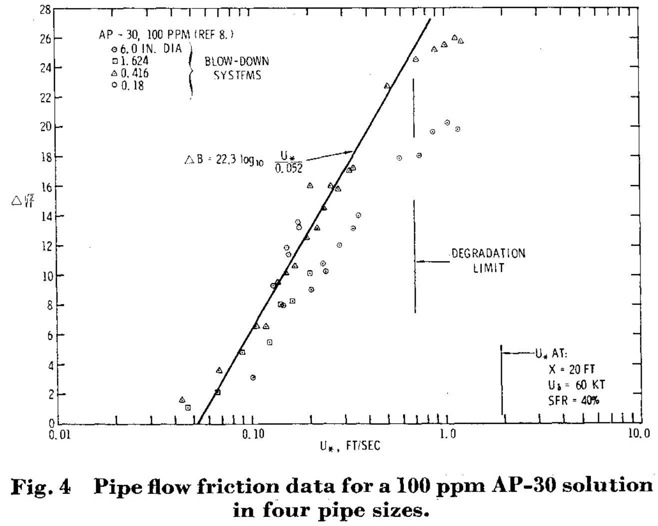

Fig. 4 Pipe flow friction data for a 100 ppm AP-30 solution in four pipe sizes.

Minimum Pipe Size Limit

In Figs. 1–4 and Fig. 7, data for the smallest pipe size fall to the right of the correlation for larger pipes. This is an indication that the friction reduction at a given friction velocity is less than in the larger pipes.

The data for the small pipes that do not fall on the general correlation also have u_{*cr} occurring before the u_* at transition to turbulence. However, there has been no clear indication that transition has been delayed by additives in pipe flow, only that there is a more gradual transition process for the case of onset of drag reduction preceding transition.

The data of Figs. 5 and 6 for the two lowest concentrations of Polyox are presented to show that Eq. (2) is valid for the lower concentrations and the pipe sizes used. There is a slight inconsistency for 5 ppm Polyox, in which the data for the 3.21 cm pipe fall to the right of the data for the 0.292-cm pipe. This appears to be due to the increased degradation in the recirculating system, discussed further in a later section. But it is clear that Polyox data can be correlated in the same way as other additives if the minimum pipe diameter limit or the degradation limit is not exceeded.

If the shift in the velocity profile, as stated in the dimensionless variables of Eq. (2) is a function of u_* only, and if the pipe radius is too small to permit development of the semilogarithmic velocity profile, the deviation between ΔB and $\Delta(2/f)^{1/2}$ should be a function of the viscous sublayer thickness and the pipe radius. Thus, there should be a critical ratio of the sublayer thickness y_L to the pipe radius R for which ΔB is no longer equal to $\Delta(2/f)^{1/2}$. This ratio can be expressed in terms of the law-of-the-wall variable as the ratio of the sublayer thickening function ΔB to the dimensionless radius $\eta_R = u_*R/\nu_w$. Therefore, the ratios $\Delta(2/f)^{1/2}/\Delta B$ and $\Delta B/\eta_R$ were calculated for all the data shown in Figs. 1–4 and 7 plotted in Fig. 8.[†] ΔB was calculated using values of α and u_{*cr} found in large pipes and the actual u_*. No data above the degradation limit were used in this figure. Although the scatter is rather large due to the random errors associated with comparing data taken with various fluids in various facilities, it is seen that the ratio of $\Delta(2/f)^{1/2}/\Delta B$ tends to become less than one at values of $\Delta B/\eta_R$ greater than

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[†] The form of the variables were also suggested from an integration of the modified Squire profile given by Granville[2] in an attempt to analytically predict the effect of transition layer thickening for the present study. The modified Squire profile was not adequate for describing the profiles or the friction factor derivation, but the form of the correlation is the same as used in Fig. 8.

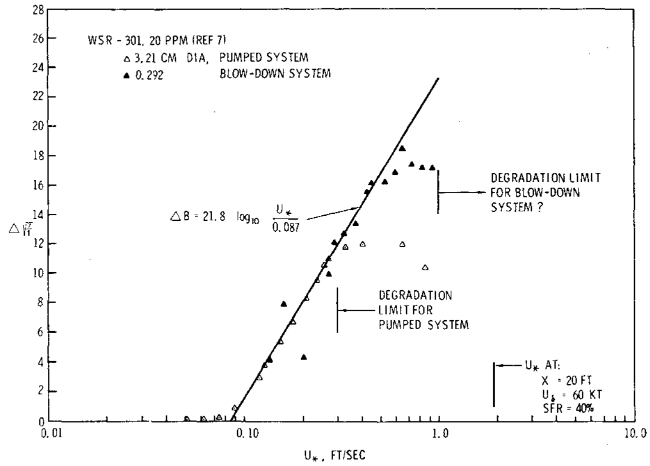

Fig. 6 Pipe flow friction data for a 20 ppm WSR-301 solution in two pipe sizes.

than 0.033. By curve fitting the data a correlation for small pipes is given, together with the limits over which it is appropriate, by

$$\Delta(2/f)^{1/2}/\Delta B = -[1.04 \log_{10}(\Delta B/\eta_R) + 0.54]$$
$$(u_* \geq u_{*cr}, \quad \Delta B/\eta_R \geq 0.033, \quad Re \geq Re_{tr}) \tag{7}$$

where Re_{tr} is the Reynolds number at transition.

Equation (7) serves as an extension of the Meyer correlation for pipe sizes less than the minimum for the semilogarithmic velocity profile. Equation (7) also indicates that the change in skin friction in small pipes can be determined from the same α and u_{*cr} used in large pipes, since those values were used to determine ΔB for the data of Fig. 8. This is further evidence that α and u_{*cr} are basic properties of a fluid.

There are threee immediate applications for Eq. (7): 1) to set the minimum diameter limits for pipe flow experiments with a given fluid, 2) to predict friction reduction for a shear layer that is below the minimum boundary-layer thickness [the appropriate friction coefficient must be used in Eq. (5) for this application], and 3) to optimize the concentration (or value of α) that is needed to achieve a given amount of friction reduction for a given boundary-layer thickness. The latter is particularly important, since several investigators have used data from capillary tubes to evaluate the optimum concentration for friction reduction.

Degradation Limit

The data in Figs. 1–7 are also useful in examining effects of degradation. Degradation, as evidenced by a levelling off or a decrease in ΔB at high values of u_*, is detected quite easily by plotting the data in this form, which also shows clearly whether degradation occurs at a given value of u_* in different pipes for a given solution. Some of the data clearly indicate degradation and some do not. (The decrease of ΔB for data taken in a single-pass system does not necessarily indicate irreversible degradation. Tests of the P-295 additive solutions in this laboratory indicate that irreversible degradation occurs when the solution is pumped for long periods, and there are numerous examples in the literature of the fragility of WSR-301. Therefore, it seems safe to assume that the falloff of ΔB at large u_* is irreversible degradation.) Values of the degradation limit on u_*, where observed, and of the highest value of u_*, where degradation was not observed, are summarized in Table 1 for all of the experiments considered.‡ Also listed are values of α and u_{*cr} for each of the solutions, as determined from pipes of sufficiently large sizes.

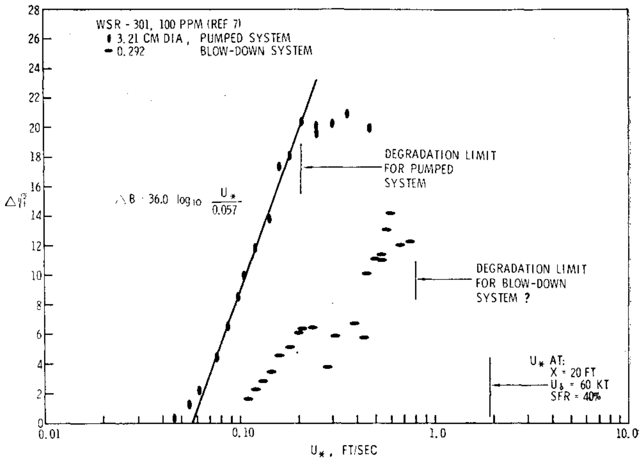

Fig. 7 Pipe flow friction data for a 100 ppm WSR-301 solution in two pipe sizes.

Also shown in Figs. 1–7 is the value of u_* for a given set of conditions on a flat plate, at a position 20 ft from the leading edge, a freestream velocity of 60 knots, and skin-friction reduction of 40% (a reduction that is not consistent with the reduction predicted for the solutions studied, but which will serve as a conservative upper limit on u_* for the length and velocity conditions). The value of u_* for these conditions is 1.93 fps.

All of the P-295 solutions show no degradation below $u_* = 1.0$ fps. The 100 ppm solution shows no degradation below $u_* = 1.5$ fps, the highest value of the experiment. The limit for the 10 ppm solution appears to be even higher, but is more difficult to detect with such a low value of α. It is concluded, then, that P-295 solutions follow the correlation, with no evidence of degradation to within one-half of the value of u_* for the example flat plate.

The AP-30 solutions, as pointed out in Ref. 8, show no degradation below $u_* = 0.7$ fps. The most interesting point of these data is that the facilities permitted duplication of the high u_* values in pipes of two different diameters, and more importantly, two different pipe lengths. The ratio of lengths of the 0.416-in. section and the 0.18-in. section was 2.4. Since Figs. 3 and 4 show degradation beginning at the same value of u_* for each solution, it can be concluded that the degradation limit is dependent on u_* and not on the length of turbulent flow for these solutions.

Degradation for the WSR-301 solutions in the 3.21-cm pipe of Virk's experiments, as shown in Figs. 5–7, occurs at u_* as low as 0.2 fps and perhaps at onset for the 5 ppm solution,

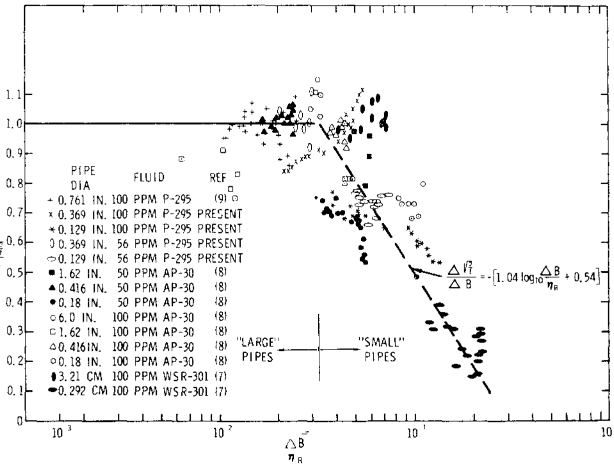

Fig. 8 A summary of the pipe flow friction data, which shows the "minimum pipe size limit" on the Meyer correlation for values of $\Delta B/\eta_R$ greater than about 0.033.

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‡ Data for 10, 18, and 31 wppm P-295 are not given in the figures as the ΔB — u_* correlation fits quite well for two pipe sizes.

as discussed earlier. However, this was a recirculating system and subject to continuous pumping and possible degradation. Data for the 0.292 cm blow-down pipe system show no degradation below $u_* = 0.8$ fps although the scatter is considerable for the 20 and 100 ppm solutions. The 5 ppm solution clearly shows no degradation up to $u_* = 0.93$ fps. Therefore, it can be concluded that the $\Delta B - u_*$ correlation is adequate up to about one-half of the value of u_* for the example flat plate. Above that limit, a constant value of ΔB is a good approximation, as suggested by Fabula.[5] It is appropriate to use the ΔB term directly in this case, since there is no evidence that degraded solutions fail to satisfy the semilogarithmic velocity profile criterion, unless the pipe diameter is too small.

There is no alternative but to duplicate the degradation conditions of the external boundary layer in the pipe flow experiments. It would seem that considerable experimentation is needed in this regard to define the degradation conditions.

Conclusions

The following conclusions are based on the preceding discussion.

1) Data for various effective drag-reducing solutions in various pipe sizes can be correlated with the friction velocity u_*, the fluid properties α and u_{*cr}, and, for the case of "small" pipes, the dimensionless pipe radius η_R. The correlation may be stated as

$$\Delta B = 0 \qquad (u_* \leq u_{*cr})$$

$$\Delta B = \alpha \log_{10} u_*/u_{*cr} \qquad (u_* \geq u_{*cr},\ \Delta B/\eta_R \leq 0.033)$$

$$\Delta(2/f)^{1/2}/\Delta B = -[1.04 \log_{10} (\Delta B/\eta_R) + 0.54]$$

$$(u_* \geq u_{*cr},\ \Delta B/\eta_R \geq 0.033,\ Re \geq Re_{tr})$$

2) It is concluded from the form of the correlation that the law-of-the-wall variables used in the basic correlation are sufficient to correlate data in small pipes, if the effect of the deviation from the semilogarithmic profile is taken into account for values of $\Delta B/\eta_R$ greater than about 0.033. This means that the correlation is sufficiently general to be applied to other types of thin boundary layers, such as external boundary layers.

3) It is also concluded from the form of the small pipe correlation that the fluid has the same effective u_{*cr} in small pipes, even when u_{*cr} occurs before transition to turbulence and even though the fluid is not drag-reducing until it is turbulent. This may give the appearance of effecting transition, but is only because of the small size of the pipes in which these low Reynolds number flows are observed.

4) The correlation is limited, for the additives studied, by degradation of the additives at high friction velocities, the order of 1 fps. After the onset of degradation, the ΔB function is approximately constant for a twofold increase in u_* past the degradation point.

5) The data suggest that degradation occurs at a constant value of u_* in pipe flow, and is independent of the pipe length.

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